IR modifications of gravity and massive gravity theories

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April 13, 2015

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Introduction

- The gravitational force is an inverse square law/infinite range force.
- This means that the gravitational force carrier is massless!
- We call this carrier "graviton".
- Our experiments show that the gravitational force is always attractive!
- So, the graviton must be a spin-2 field. (Weinberg)
- Einstein, identified this spin-2 filed with the metric of a curved space-time.

Introduction

- A massless spin-2 particle has 2 helicity states/ 2 d.o.f.
- Matter content of the universe will alter the geometry of space-time.

$$G_{\mu\nu} = 8\pi T_{\mu\nu}.$$

- The above equation is the Einstein's general relativity.
- What should we modify GR?
- GR can only explain the solar system physics correctly!
- This is somehow an intermediate scales, in the universe!

UV modification

- For very small scales/UV limit, one should replace GR with a quantum theory of gravity!
- One needs a QFT of gravity/QFT for spin-2 field.
- The theory has many difficulties; can't be considered as a well-defined theory; non-renormalizability, definition of vacuum state,...
- Other attempts to build such a theory is
  1. String theory; replacing point particles with strings...
     Green, Gross, Schwarz, Witten, Vafa, Garousi,...
  2. Loop quantum gravity; discarding the continuum of space-time at UV...
     Ashtekar, Rovelli, Tiemann,...
  3. Local dynamical triangulations; triangulate the fabric of space-time!
     Loll, Ambjorn, Jurkiewicz,...

IR modification

- At large scales/IR limit, deviations from GR is observed, such as the accelerated expansion of the Universe.
- Considering usual attractive gravitational force, the expansion of the universe should be decelerating!
- The accelerated expansion is related to something like a repulsive gravitational force!!
- This accounts to modify GR at IR; the IR modification of GR.

IR modification

- Cosmologists add some dark energy to the universe, to explain this expansion! Dark energy has negative pressure!
- This can be achieved by adding a constant to Einstein's GR!

$$G_{\mu\nu} + \Lambda g_{\mu\nu} = 8\pi T_{\mu\nu}.$$

- Λ is the cosmological constant!.
- One can go further and replace this constant by a scalar field, the so-called quintessense, or after using in inflation, the inflaton.

IR modification

- Theoretical physicists, try to modify the geometry of space-time to obtain an accelerated expanding universe.
- Changing the Einstein general relativity results in the modified gravity theories!

$$S = \int d^4x \sqrt{-g} R.$$

- A bunch of modified theories of gravity exists!!
  1. Going to higher dimensions.
  2. Adding higher derivative self-interactions term.
  3. Enriching the geometry of space-time/ Weyl, Cartan,...
  4. Changing the force carrier!
  5. etc.

Massive gravity

- Instead of producing repulsive gravitational force, one can weaken the strength of attractive gravity.
- Giving mass to graviton, will help us!
- A massive force carrier has a Yukawa type potential

$$V(r) \propto \frac{e^{-mr}}{r}.$$

- So, in principle, massive graviton can explain the accelerated expansion of the universe.
- Up to boundary of the universe, about

$$r \sim H_0^{-1} \sim 1.5 \times 10^{10} \text{ lyr} \sim 1.4 \times 10^{26}\ m$$

graviton should be massless!

IR modification

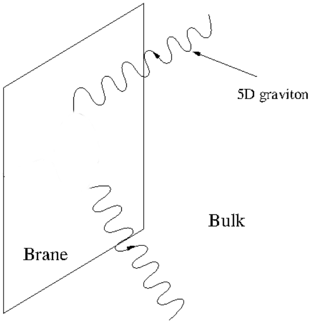
- The potential should deviate from $\frac{1}{r}$ law only at the boundary of the universe, giving the graviton mass

$$m_g \sim 10^{-33}\ eV \qquad \text{or} \qquad m_g \sim 2 \times 10^{-39} m_e$$

- Graviton mass is very very small! But produce very large modification!
- Change d.o.f of the theory from 2 to 5.
- The resulting theory is called Massive Gravity.
- Two major categories of MG exist
  1. Soft massive gravities; Write a theory which effectively produce massive graviton; DGP theory, Cascading gravities,...
  2. Hard massive gravities; Write a theory directly for a massive spin-2 field; Fierz-Pauli type theories, dRGT theory,...

DGP model
An example of soft theory

- One can think that our 4 dimensional universe, called the brane, lives in a 5 dimensional space-time called, the bulk.
- All standard model forces (electromagnetism, weak and strong nuclear forces) live in 4 dimensions. All daily experiments will be satisfied!



DGP model

- Gravity can propagate to fifth dimension! So, graviton is a 5 dimensional object!
- A massless graviton in 5D has 5 d.o.f.
- So, the effective 4 dimensional graviton should be massive.
- Consider the action of the form

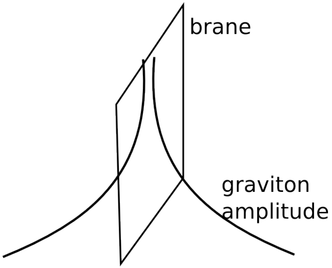
$$S = M_\star^3 \int d^4x dy \sqrt{-{}^5g}\ {}^5R + M_p^2 \int d^4x \sqrt{-{}^4g}\ {}^4R + \int d^4x \sqrt{-{}^4g}\mathcal{L}_m.$$

- The form of propagator in momentum space is

$$\tilde{G}(p, y) = \frac{1}{2M_\star^3 p + M_p^2 p^2} e^{-p|y|}.$$

DGP model

- There exists a cross-over scale $r_c = \frac{M_p^2}{M_\star^3}$.
- For $r \ll r_c$ graviton is 4 dimensional.
- For $r \gg r_c$ graviton is 5 dimenstional. In 4D viewpoint the graviton at large scales is massive.



- DGP model can explain the hierarchy problem.
- Healthy models exist only in the flat background!

Fierz-Pauli action

- The FP action

$$S = \int d^4x \Big(-\frac{1}{2}\partial_\lambda h_{\mu\nu}\partial^\lambda h^{\mu\nu} + \partial_\mu h_{\nu\lambda}\partial^\nu h^{\mu\lambda} - \partial_\mu h^{\mu\nu}\partial_\nu h + \frac{1}{2}\partial_\lambda h \partial^\lambda h - \frac{1}{2} m^2 (h_{\mu\nu}h^{\mu\nu} - h^2) \Big).$$

- FP mass term is the unique ghost and tachyon free mass term for a spin-2 field. (Fierz and Pauli 1939)
- There is no general covariance (preferred background).

Fierz-Pauli action

- The $m = 0$ case is invariant under the linearized general coordinate transformations

$$\delta h_{\mu\nu} = \partial_\mu \xi_\nu + \partial_\nu \xi_\mu.$$

  hence, 2 d.o.f.

- The massive case breaks the above symmetry, and so acquire 3 more d.o.f.
- -1 in the mass term is necessary for the absence of ghost.
- Term like $-\frac{1}{2}m^2(h_{\mu\nu}h^{\mu\nu} - (1-a)h^2)$ with $a \neq 0$ will describe a scalar ghost with mass $m_g^2 = \frac{3-4a}{2a}m^2$.
- For small a, the ghost mass squared behaves like $\frac{1}{a}$. Becomes infinite in the limit $a \to 0$. A non-dynamical ghost!

The vDVZ problem

- The massless graviton propagator in the momentum space is (after fixing the gauge redundancies)

$$\mathcal{D}_{\alpha\beta,\sigma\lambda} = \frac{-i}{p^2}\left[\frac{1}{2}\left(\eta_{\alpha\sigma}\eta_{\beta\lambda} + \eta_{\alpha\lambda}\eta_{\beta\sigma}\right) - \frac{1}{2}\eta_{\alpha\beta}\eta_{\sigma\lambda}\right],$$

- This is the standard inverse square law of the gravitational filed.
- The massive propagator on the other hand

$$\mathcal{D}_{\alpha\beta,\sigma\lambda} = \frac{-i}{p^2+m^2}\left[\frac{1}{2}\left(P_{\alpha\sigma}P_{\beta\lambda} + P_{\alpha\lambda}P_{\beta\sigma}\right) - \frac{1}{3}P_{\alpha\beta}P_{\sigma\lambda}\right],$$

with

$$P_{\alpha\beta} \equiv \eta_{\alpha\beta} + \frac{p_\alpha p_\beta}{m^2}.$$

The vDVZ problem

- Ignoring the ill behavior of $P_{\alpha\beta}$ in the limit $m \to 0$, there is a more serious problem in the above propagators.
- There is a difference in coefficient for the last term, even as $m \to 0$.
- It is $1/2$ vs. $1/3$.
- This is the first appearance of the vDVZ discontinuity.
- One can cure the above illness with a brilliant trick *a la* Stuckelberg.

Non-linear massive gravity

- This discontinuity can be cured by non-linear interactions! (Vainshtein 1972)
- The simplest non-linear massive gravity action is

$$S = \frac{1}{2\kappa^2}\int d^4x \left[(\sqrt{-g}R) - \sqrt{-g^0}\frac{1}{4}m^2 g^{(0)\mu\alpha} g^{(0)\nu\beta}\left(h_{\mu\nu}h_{\alpha\beta} - h_{\mu\alpha}h_{\nu\beta}\right)\right].$$

- There is no general covariance in the theory.
- These non-linear terms give rise to a ghost in massive gravity! (Boulware and Deser 1972)
- So the problem is how one can construct a general covariant mass term for the spin-2 particle $g_{\mu\nu}$?

dRGT massive gravity

- One of the general covariant versions of Fierz-Pauli action is

$$S = -M_p^2 \int d^4x\sqrt{-g}\,R(g) + 2M_p^2 m^2 \int d^4x\sqrt{-g}\,(e_2(\mathcal{K}) + \alpha_3 e_3(\mathcal{K}) + \alpha_4 e_4(\mathcal{K}))\,.$$

where

$$e_2(\mathcal{K}) = [\mathcal{K}]^2 - [\mathcal{K}^2]\,,$$
$$e_3(\mathcal{K}) = [\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\,,$$
$$e_4(\mathcal{K}) = [\mathcal{K}]^4 - 6[\mathcal{K}^2][\mathcal{K}]^2 + 8[\mathcal{K}^3][\mathcal{K}] + 3[\mathcal{K}^2]^2 - 6[\mathcal{K}^4]\,,$$

and

$$\mathcal{K}^\mu_\nu(g,\phi^a) = \delta^\mu_\nu - \sqrt{g^{\mu\alpha}f_{\alpha\nu}}$$

dRGT massive gravity

- Equation of motion

$$G_{\mu\nu} + m^2 X_{\mu\nu} = \kappa T_{\mu\nu} \,,$$

with

$$X_{\mu\nu} = -\frac{1}{2}\Big[\mathcal{K} g_{\mu\nu} - \mathcal{K}_{\mu\nu} + \alpha\left(\mathcal{K}^2_{\mu\nu} - \mathcal{K}\mathcal{K}_{\mu\nu} + \frac{1}{2} g_{\mu\nu}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)\right)$$
$$+ 6\beta\Big(\mathcal{K}^3_{\mu\nu} - \mathcal{K}\mathcal{K}^2_{\mu\nu} + \frac{1}{2}\mathcal{K}_{\mu\nu}\left([\mathcal{K}]^2 - [\mathcal{K}^2]\right)$$
$$- \frac{1}{6} g_{\mu\nu}\left([\mathcal{K}]^3 - 3[\mathcal{K}][\mathcal{K}^2] + 2[\mathcal{K}^3]\right)\Big)\Big] \,.$$

where

$$\alpha_3 = (\alpha - 1)/3, \quad \alpha_4 = -\beta/2 + (\alpha + 1)/12.$$

dRGT massive gravity

- There is no flat and closed FRW solution for the model! (D'Amico *et al.* 2011)
- But there is an open FRW solution (Mukohyama *et al.* 2011)

$$
\begin{aligned}
3H^2 - \frac{3|K|}{a^2} &= \rho_m + c_\pm m_g^2, \\
-\frac{2\dot{H}}{N} - \frac{2|K|}{a^2} &= \rho_m + p_m,
\end{aligned}
$$

where

$$
\begin{aligned}
c_\pm \equiv & -\frac{1}{(\alpha_3+\alpha_4)^2}\left[1+\alpha_3 \pm \sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}\right] \\
& \times \left[1+\alpha_3^2-2\alpha_4 \pm (1+\alpha_3)\sqrt{1+\alpha_3+\alpha_3^2-\alpha_4}\right].
\end{aligned}
$$

Some difficulties

- dRGT massive gravity suffers from the appearance of superluminal modes, in some specific cosmological solutions.
- One can cure this problem by adding an extra scalar field σ with dilatation symmetry:

$$g_{\mu\nu} \to e^{-2\alpha} g_{\mu\nu}, \qquad \sigma \to \sigma - M_{Pl}\alpha.$$

- α is constant.
- The above symmetry is global.

Theorem

(Gruzinov) All massive gravity theories derived from Fierz-Pauli action suffers from ghost and/or superluminal instability.

Thanks for your attention.

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