Dark Energy OR Baryonic Matter?

Zahra Haghani

Damghan University

April 20, 2015

The Einstein-Hilbert action

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}\,(R-2\Lambda) + \mathcal{S}_m,$$

The field equations

$$G_{\mu\nu} + g_{\mu\nu}\Lambda = 8\pi G T_{\mu\nu}.$$

Identically we have

$$\nabla^\mu G_{\mu\nu} = 0 \quad \Rightarrow \quad \nabla^\mu T_{\mu\nu} = 0$$

Modified Gravity Theories:

- Higher dimensional gravity.
  RS 1999, DGP 2000, STM 1999, Loveloke 1971, ...
- Higher derivative gravity
  $f(R)$, $f(R,T)$, $f(G)$, ...
- Non-Riemannian geometry
  Weyl 1918, Cartan 1922, ...
- Tensor & ... gravity
  Brans-Dicke 1961, TeVeS 2004, STVG 2006, ...
- ...

Lovelock Lagrangian

- Metric theory of gravity in D dimensional space-time.
- Results in second order equation of motion.
- The field equations is conserved.

$$\mathcal{L} = \sqrt{-g}\ \sum_{n=0}^{t} \alpha_n\ \mathcal{R}^n,$$

where

$$\mathcal{R}^n = \frac{1}{2^n}\delta^{\mu_1\nu_1\ldots\mu_n\nu_n}_{\alpha_1\beta_1\ldots\alpha_n\beta_n}\prod_{r=1}^{n} R^{\alpha_r\beta_r}{}_{\mu_r\nu_r}, \quad \delta^{\mu_1\nu_1\ldots\mu_n\nu_n}_{\alpha_1\beta_1\cdots\alpha_n\beta_n} = \frac{1}{n!}\delta^{\mu_1}_{[\alpha_1}\delta^{\nu_1}_{\beta_1}\cdots\delta^{\mu_n}_{\alpha_n}\delta^{\nu_n}_{\beta_n]}.$$

$$\mathcal{L} = \sqrt{-g}\ \left(\alpha_0 + \alpha_1 R + \alpha_2\left(R_{\alpha\beta\mu\nu}R^{\alpha\beta\mu\nu} - 4R_{\mu\nu}R^{\mu\nu} + R^2\right) + \ldots\right),$$

Brans-Dicke Theory

- Respects to the Mach Principle.
- The gravitational interaction is mediated by a scalar field as well as the tensor field of general relativity.
- The gravitational constant G is not presumed to be constant.

$$S = \frac{1}{16\pi}\int d^4x\sqrt{-g}\left(\phi R - \omega\frac{\partial_\mu\phi\partial^\mu\phi}{\phi}\right) + \mathcal{S}_m.$$

- The field equations

$$\Box\phi = \frac{8\pi}{3+2\omega}T,$$
$$G_{\mu\nu} = \frac{8\pi}{\phi}T_{\mu\nu} + \frac{\omega}{\phi^2}(\partial_\mu\phi\partial_\nu\phi - \frac{1}{2}g_{\mu\nu}\partial_\rho\phi\partial^\rho\phi) + \frac{1}{\phi}(\nabla_\mu\nabla_\nu\phi - g_{\mu\nu}\Box\phi).$$

- For $T \neq 0$, in the limit $\omega \to \infty$, BD theory tends to GR.

Higher derivative gravity

- Frist attemp in this way was done by Weyl to unify gravity and electromagnetic forces.
- In the low-energy regime, adding higher order derivative terms and non-minimal couplings between matter and gravity can lead to a renormalizable theory at the one-loop level.
- Self-consistent inflationary model.
- To explain dark energy and dark matter without exotic matter and energy.
- There is no *a priori* reason to restrict the gravitational action to be the Einstein-Hilbert action with minimal coupling.

$f(R)$ gravity:

- The action

$$S = \frac{1}{16\pi G}\int d^4x\sqrt{-g}f(R) + \mathcal{S}_m.$$

- The field equations

$$f'(R)R_{\mu\nu} - \frac{1}{2}f(R)g_{\mu\nu} + \left[g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu\right]f'(R) = 8\pi G T_{\mu\nu},$$

- Rewritten the field equations

$$G_{\mu\nu} = 8\pi G_{eff}T_{\mu\nu} + T^{curv}_{\mu\nu}$$

$$G_{eff} = \frac{G}{f'}, \qquad T^{curv}_{\mu\nu} = \frac{1}{2}g_{\mu\nu}(\frac{f}{f'} - R) - \frac{1}{f'}(g_{\mu\nu}\Box - \nabla_\mu\nabla_\nu)f'$$

- There is a correspondence between $f(R)$ gravity and Brans-Dicke theory.
- This correspondence shows that there is a non-minimal coupling between matter and geometry.
- We add a non-minimal coupling between matter and geometry by hand.

- **What conditions one should impose?**

Non-minimal coupling

- In the full Lagrangian, the matter fields should couple to curvature minimally: The principle of minimal coupling.
- One can generalize the above statement to obtain a weaker principle: In the matter Lagrangian, matter fields should not couple to the curvature.
- The first attempt to couple matter and geometry in a non-minimal way was carried out by H. Goenner [1].

Non-minimal coupling

- In the full Lagrangian, the matter fields should couple to curvature minimally: The principle of minimal coupling.
- One can generalize the above statement to obtain a weaker principle: In the matter Lagrangian, matter fields should not couple to the curvature.
- The first attempt to couple matter and geometry in a non-minimal way was carried out by H. Goenner [1].

His postulates are:

- The field equation should follow from the variational principle.
- Vacuum solutions of the theory should be identical to GR vacuum solutions.
- The Newtonian limit should hold.
- Form of the non-minimal couplings should be independent of the specific material of the universe.

Non-minimal coupling

- In the full Lagrangian, the matter fields should couple to curvature minimally: The principle of minimal coupling.
- One can generalize the above statement to obtain a weaker principle: In the matter Lagrangian, matter fields should not couple to the curvature.
- The first attempt to couple matter and geometry in a non-minimal way was carried out by H. Goenner [1].

He had suggested the Lagrangian:

$$\mathcal{L} = \sqrt{-g}\left[\frac{1}{16\pi G}R + (1 + \phi(R^{\alpha}_{\beta\rho\sigma}))L_m\right],$$

and considered the case $\phi(R^{\alpha}_{\beta\rho\sigma}) = \alpha R$.

- One can generalize the above Lagrangian [2] to

$$\mathcal{L} = \sqrt{-g} f(R, L_m).$$

- But why L_m? We can equivalently couple the curvature directly to the energy-momentum tensor $T_{\mu\nu}$ of the matter!
- The first attempt was done by Paplawski [3] with the Lagrangian

$$\mathcal{L} = \sqrt{-g}\left[R + \Lambda(T)\right] + \mathcal{L}_m.$$

- One can couple the Ricci scalar to the trace of energy-momentum tensor in a more general way:

$$\mathcal{L} = \sqrt{-g} f(R, T) + \mathcal{L}_m.$$

The action

- The action for our model is

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} f\left(R, T, R_{\mu\nu} T^{\mu\nu}\right) + \int d^4x \sqrt{-g} L_m,$$

- $T_{\mu\nu}$ is the energy-momentum tensor of ordinary matter,

$$T_{\mu\nu} = -\frac{2}{\sqrt{-g}} \frac{\delta\left(\sqrt{-g} L_m\right)}{\delta g^{\mu\nu}}.$$

- T is the trace of $T_{\mu\nu}$!

Field equations

- The field equations can then be obtained as

$$
\begin{aligned}
&(f_R - f_{RT}L_m)G_{\mu\nu} \\
&+ \left[\Box f_R + \frac{1}{2}Rf_R - \frac{1}{2}f + f_T L_m + \frac{1}{2}\nabla_\alpha\nabla_\beta\left(f_{RT}T^{\alpha\beta}\right)\right] g_{\mu\nu} \\
&- \nabla_\mu\nabla_\nu f_R + \frac{1}{2}\Box\left(f_{RT}T_{\mu\nu}\right) + 2f_{RT}R_{\alpha(\mu}T_{\nu)}^{\ \alpha} - \nabla_\alpha\nabla_{(\mu}\left[T^\alpha_{\ \nu)}f_{RT}\right] \\
&- \left(f_T + \frac{1}{2}f_{RT}R + 8\pi G\right)T_{\mu\nu} - 2\left(f_T g^{\alpha\beta} + f_{RT}R^{\alpha\beta}\right)\frac{\partial^2 L_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}} = 0.
\end{aligned}
$$

- The last term is zero for a perfect fluid and scalar field.
  This term is non-zero for the electromagnetic field

$$\frac{\partial^2 L_m}{\partial g^{\mu\nu}\partial g^{\alpha\beta}} = -\frac{1}{2}F_{\mu\alpha}F_{\nu\beta}.$$

- One can rewrite the field equation as

$$G_{\mu\nu} = 8\pi G_{eff} T_{\mu\nu} - \Lambda_{eff} g_{\mu\nu} + T^{eff}_{\mu\nu},$$

with

$$G_{eff} = \frac{G + \frac{1}{8\pi}\left(f_T + \frac{1}{2} f_{RT} R - \frac{1}{2}\Box f_{RT}\right)}{f_R - f_{RT} L_m},$$

$$\Lambda_{eff} = \frac{2\Box f_R + R f_R - f + 2 f_T L_m + \nabla_\alpha \nabla_\beta (f_{RT} T^{\alpha\beta})}{2(f_R - f_{RT} L_m)},$$

and

$$T^{eff}_{\mu\nu} = \frac{1}{f_R - f_{RT} L_m}\Bigg\{\nabla_\mu \nabla_\nu f_R - \nabla_\alpha f_{RT} \nabla^\alpha T_{\mu\nu} - \frac{1}{2} f_{RT} \Box T_{\mu\nu} - 2 f_{RT} R_{\alpha(\mu} T_{\nu)}^{\ \alpha} + \nabla_\alpha \nabla_{(\mu} \left[T^\alpha_{\ \nu)} f_{RT}\right] + 2\left(f_T g^{\alpha\beta} + f_{RT} R^{\alpha\beta}\right) \frac{\partial^2 L_m}{\partial g^{\mu\nu} \partial g^{\alpha\beta}}\Bigg\}.$$

What happens to a massive test particle?

- Using the expression for $\nabla_\mu T^{\mu\nu}$ the geodesic equation in this model can be written as

$$\frac{d^2x^\lambda}{ds^2} + \Gamma^\lambda{}_{\mu\nu} u^\mu u^\nu = f^\lambda,$$

with

$$\begin{aligned} f^\lambda &= \frac{1}{\rho + p} \Big[\left(f_T + R f_{RT}\right) \nabla_\nu \rho - \left(1 + 3 f_T\right) \nabla_\nu p \\ &- (\rho + p) f_{RT} R^{\sigma\rho} \left(\nabla_\nu h_{\sigma\rho} - 2 \nabla_\rho h_{\sigma\nu}\right) \\ &- f_{RT} R_{\sigma\rho} h^{\sigma\rho} \nabla_\nu \left(\rho + p\right) \Big] \frac{h^{\lambda\nu}}{1 + 2 f_T + R f_{RT}}. \end{aligned}$$

- There is an extra force on a test particle in this theory.
- So, the violation of strong equivalence principle!

An extra acceleration?

- The role of extra acceleration can be obtained by considering the action

$$S_p = \int L_p \, ds = -\int \sqrt{Q} \sqrt{-g_{\mu\nu} u^\mu u^\nu} \, ds,$$

with

$$f^\lambda = (g^{\nu\lambda} + u^\nu u^\lambda) \nabla_\nu \ln \sqrt{Q},$$

- In the present model we have

$$f^\lambda = \frac{F}{\rho} h^{\lambda\nu} \nabla_\nu \rho,$$

where

$$F = \frac{f_T + f_{RT}(R - R_{\alpha\beta} h^{\alpha\beta})}{1 + 2f_T + R f_{RT}}.$$

- $\sqrt{Q}$ can be expanded around the background energy-density ρ_0

$$\sqrt{Q} \approx 1 - F_0 + \frac{F_0}{\rho_0}\rho,$$

with $F_0 \equiv F(\rho_0)$.

- One can obtain the extra acceleration as

$$\vec{a} = -\vec{\nabla}\phi - \vec{\nabla}U(\rho) = \vec{a}_N + \vec{a}_E,$$

- The extra acceleration is

$$\vec{a}_E(\rho) = -\vec{\nabla}U(\rho) = \frac{F_0}{\rho_0}\vec{\nabla}\rho.$$

- The extra acceleration depends on the gradient of the matter energy-density.

The generalized Poisson equation

- Suppose that the Universe is filled by a non-relativistic matter i.e. $p \ll \rho$.
- The weak field expansion of the theory can be obtain by setting $g_{00} = -(1+2\phi)$ and then $R = -2R_{00} = -2\nabla^2\phi$.
- ϕ is the Newtonian potential.

- The generalized Poisson equation is

$$\begin{aligned} \nabla^2\phi &= \frac{1}{2(f_R - 2\rho f_{RT})}\Big[8\pi G\rho + 3\nabla^2 f_R - 3\rho f_T \\ &- 2f + \nabla(3f_R + \rho f_{RT})\cdot\nabla\phi\Big]. \end{aligned}$$

An example: $f = R + \alpha R_{\mu\nu} T^{\mu\nu}$

- The flat FRW line element:

$$ds^2 = -dt^2 + a^2(t)(dx^2 + dy^2 + dz^2)$$

- Hubble parameter:

$$H = \frac{\dot{a}}{a}$$

- The field equations with perfect fluid as the matter content of the universe are

$$3H^2 = \frac{\kappa}{1 - \alpha\rho}\rho + \frac{3}{2}\frac{\alpha}{1 - \alpha\rho}H(\dot{\rho} - \dot{p}),$$

and

$$2\dot{H} + 3H^2 = \frac{2\alpha}{1 + \alpha p}H\dot{\rho} - \frac{\kappa p}{1 + \alpha p} + \frac{1}{2}\frac{\alpha}{1 + \alpha p}(\ddot{\rho} - \ddot{p}).$$

- For dust with the equation of state $p=0$ and by assuming $\alpha\rho \ll 1$, de Sitter solution $H = H_0 = \text{constant}$ exists with

$$\rho(t) = e^{\frac{1}{2}H_0(t-t_0)}\left\{\frac{\sqrt{\alpha}\,(2\rho_{01} - H_0\rho_0)}{\sqrt{\alpha H_0^2 - 8\kappa}}\sinh\left[\frac{\sqrt{\alpha H_0^2 - 8\kappa}}{2\sqrt{\alpha}}(t-t_0)\right] + \rho_0\cosh\left[\frac{\sqrt{\alpha H_0^2 - 8\kappa}}{2\sqrt{\alpha}}(t-t_0)\right]\right\},$$

  where we have used the initial conditions $\rho(t_0) = \rho_0$, and $\dot{\rho}(t_0) = \rho_{01}$.

- In this case, weak energy condition holds for $\alpha > 0$ [5].

Conclusions

- Non-minimal coupling between matter and geometry seems to be a viable generalization of $f(R)$ theories.
- One can couple the geometry to the matter Lagrangian or to the energy-momentum tensor.
- In $f(R,T)$ theories, the interaction between geometry and electromagnetic field is not considered. The addition of $R_{\mu\nu}T^{\mu\nu}$ coupling can solve this problem.
- This model can also explain the late time acceleration of the Universe.
- The extra force could explain the properties of the galactic rotation curves without resorting to the dark matter hypothesis.

References

- H. Goenner, Found. Phys. 14, 865 (1984).
- T. Harko and F. S. N. Lobo, Eur. Phys. J. C 70, 373 (2010).
- N. Paplawski, arXiv:gr-qc/0608031 (2006).
- Z. Haghani, T. Harko, F. Lobo, H. R. Sepangi and S.Shahidi, Phys. Rev. D 88, 044023 (2013).
- M. Sharif, M. Zubiar, JHEP 1312 079 (2013).